Available online at www.sciencedirect.com

SCIENCE DIRECT®

Human Movement Science 23 (2004) 621–641

HUMAN MOVEMENT SCIENCE

www.elsevier.com/locate/humov

Evaluating the coordination dynamics of handwriting

Sylvie Athènes [a,*], Isabelle Sallagoïty [b], Pier-Giorgio Zanone [b], Jean-Michel Albaret [b]

[a] *Centre d'Etudes de la Navigation Aérienne, 7 Avenue Edouard Belin, BP 4005, 31005 Toulouse Cedex, France*

[b] *EA 3691 "Laboratoire Adaptation, Perceptivo-Motrice et Apprentissage", UFR STAPS, Université Paul Sabatier, 118 route de Narbonne, 31062 Toulouse Cedex 4, France*

Abstract

This study aims to test the hypothesis that handwriting is governed by the dynamics of non-linear coupled oscillators. Accordingly, its first goal is to identify preferred, basic graphic shapes corresponding to spontaneously stable combinations of the two frequency-locked oscillatory *x–y* components of the trajectories. Six participants were required to produce 26 ellipsoids of varying eccentricities and orientations presented consecutively on a graphic tablet. These shapes corresponded to a systematic manipulation of the relative phase and the relative amplitude of the oscillators by a constant step. Results showed that among those, only eight ellipsoids were produced in a spontaneous and stable fashion. They were characterized by attraction of nearby shapes, and by a higher accuracy and velocity. Alike all periodic motion, graphic skills, hence handwriting, exhibit preferred coordination patterns, which can be ascribed to the non-linear coupling of two oscillators.

Keywords: Dynamical system; Self-organization; Dynamic pattern theory; Graphonomics

* Corresponding author. Tel.: +33 5 62 25 95 91.
E-mail address: athenes@cena.fr (S. Athènes).

0167-9457/$ - see front matter
doi:10.1016/j.humov.2004.10.004

1. Introduction

Handwriting can be defined as the visible trace of a spoken language. The relationship between the visible signs and the audible signs can be quite different from one language to another, going from systems where one sign can encode a whole word to alphabetical systems where each sign encodes a basic sound. The appearance of the signs themselves varies greatly, with language being written either from left to right, or right to left, or even top to bottom. In spite of this staggering diversity, there are some commonalities shared by all handwriting systems. For example, their communication function imposes that, within each language, some – presumably important – features vary systematically less than others, lest the reader spends too much time and effort deciphering the handwritten trace. Furthermore, regardless of the language, the signs are semantically meaningful shapes that are always traced by the – usually right – hand of a human being. Acting together, these two constraints, trace legibility and human motor properties, effectively reduce the "creativity" of the writer. Our tenet is that handwriting systems, having evolved over many centuries, are a working compromise between a small number of fast and comfortable human arm/wrist/finger movements producing stable shapes and the necessity to have a large enough variety of shapes easily recognizable and differentiable. Indeed, shapes do look different across different languages. Yet, the underlying movements cannot be quite as different in spite of the variability induced by the medium, whether it be a pencil or a brush, for example.

Comparing different handwriting systems (Arabic, Hebrew, Chinese, and English among others), Van Sommers (1984) showed that basic stroke directions are fairly similar: strokes towards the bottom-right and the bottom-left (and right, and top-right to a certain extent) are most frequent, whereas strokes towards the top and top left are virtually non-existent. Further arguments come from studies by language historians reporting similar evolution of the shapes for different languages. For example, Irigoin (1990) describes the evolution of Ancient Greek from capital letters to cursive handwriting: angles become curves, letters are simplified and linked, and differences between similar shapes are enhanced (e.g., Σ became C and not ε as expected, because it would have been too close to the already existing e). Summing up his observations, this author contends that every writing system is a structure that reaches a more or less stable equilibrium between two conflicting tendencies: a tendency to simplification with a reduced number of basic strokes, easy to execute, combined to form letters, and a tendency to differentiation in order to avoid similar looking letters, which would decrease legibility and slow down the reading. If relative stability is an important notion to understand basic shapes in handwriting, careful manipulation of one or the other tendency (simplification versus differentiation) should then result in lawful changes in the relative frequency of the different shapes composing handwriting. The present paper, a first attempt at a systematic exploration of the handwriting relative stability in the face of constraints acting on movement execution, addresses exclusively the basis for the tendency to simplification.

Handwriting is a complex motor skill that requires the coordination of numerous structures and components in order to produce a succession of graphic shapes of varying size and direction. In particular, the pen trajectory results from the coordination of the arm-finger system that, alone, involves more than ten biomechanical degrees of freedom. Several studies (e.g., Teulings, 1996), however, showed that the final outcome might be reduced to two single degrees of freedom. The first corresponds to the flexion–extension motion of the finger joints, while the second involves adduction–abduction movements of the wrist. Thus, handwriting may eventually be resumed through two generators of motion: anterio-posterior oscillations (finger joints), lateral oscillations (wrist joint), with an added translational motion to the right (elbow and shoulder joints) (Teulings, Thomassen, & Maarse, 1989). The latter component has only a marginal influence on the production of distinct letters (Thomassen & Meulenbroek, 1998) and is therefore neglected in the modeling of handwriting. Hollerbach (1981) proposed a formalization of handwriting based on sinusoidal motion. The model basically assumes that each antagonist muscle pair behaves as a harmonic mass-spring system. Handwriting is thus generated through the combined action of a pair of oscillators set in an orthogonal fashion. Both oscillatory components follow the equation:

$$\begin{aligned} x(t) &= A_x \cos(\omega_x(t - t_0) + \phi_x) \\ y(t) &= A_y \cos(\omega_y(t) + \phi_y) \end{aligned} \tag{1}$$

where A_x and A_y are each oscillator's amplitude, ω_x and ω_y their eigenfrequency, and ϕ_x and ϕ_y their phase. The produced letters result then from the combined oscillation in the horizontal and vertical directions, due to the action of the anterio-posterior and lateral generators, respectively. The addition of a translational motion from left to right at a constant speed distinguishes separate letters spatially. Geometrically speaking, combining two sinusoidal functions with equal eigenfrequencies ($\omega_x = \omega_y$) gives rise to ellipses of varying eccentricities and orientations, depending on the relative amplitude (RA = A_y/A_x) and the relative phase (RP = $\phi_x - \phi_y$) between the oscillations. The model assumes that the oscillators are merely combined to produce a given output, so that the totality of the parameters to be implemented by the oscillators must be ready before handwriting starts in order to perform the expected graphic trajectory. A benefit of the model is that only a few parameters are necessary to realize the entire set of all graphic shapes performed in handwriting.

Along with many studies striving to evidence basic behavioral units for motor programs (Hulstijn & van Galen, 1983; Teulings, Thomassen, & van Galen, 1986; Van Galen, 1991), the above model addresses neither the biomechanical properties of the hand nor the preferred tendencies existing in graphic skills (Meulenbroek & Thomassen, 1991; Van Sommers, 1984). Typically, this type of model does not predict how variations in the production of a letter may occur in such a stereotyped fashion when the level of constraint on behavior such as speed is increased (Van der Plaats & van Galen, 1990). However, more recent studies (Dounskaia, van Gemmert, & Stelmach, 2000) argued that these preferences stem from the coordination between finger and wrist movements and from the properties of interjoint biomechanics. A useful feature of a model would then be the prediction of how handwriting deteriorates with increasing level of constraint.

In light of the aforementioned results, it seems reasonable to examine handwriting behavior in the framework of the so-called dynamical systems approach. Our main claim is that the observed behavioral preferences in graphic skills reflect intrinsic tendencies pertaining to underlying coordination dynamics. Like most periodic motor behavior, graphic skills may be conceived of as the outcome of *non-linear* coupled oscillators. In the view of a dynamical systems approach, behavior results from the self-organization of the system's many degrees of freedom (Kelso & Schöner, 1987). Concepts such as collective variable (also known as order parameter), multistability, loss of stability, and the mathematical tools of dynamical systems proved to provide a fruitful framework for understanding coordination in biological systems, in particular motor behavior (Kelso, 1995). In a seminal work on bimanual coordination, Kelso (1984) showed that limbs may be considered as biological oscillators, so that their coupling brings about only two coordination patterns: in-phase and anti-phase. Such patterns are coined attractors, because irrespective of its initial state, the system will eventually return to such preferred patterns characterized by a larger stability. This bistable coordination dynamics governs then the behavioral patterns that the system may exhibit spontaneously. A benefit is that all such patterns, as well as the transitions among them, are described univocally by one single collective variable: the relative phase between the oscillating limbs. The equation of motion of relative phase captures all the coordinated behaviors observed macroscopically, without consideration of the components themselves, which reduces substantially the system's relevant degrees of freedom. Formally, the time course of relative phase has been modeled by Haken, Kelso, and Bunz (1985) in terms of non-linear coupled oscillators. These tendencies to synchronize in-phase and anti-phase have been reported in multilimb coordination (Kelso & Jeka, 1992), inter-individual coordination (Schmidt, Carello, & Turvey, 1990), as well as in trajectory formation (Buchanan, Kelso, & Fuchs, 1996; Buchanan, Kelso, and Guzman, 1997; de Guzman, Kelso, & Buchanan, 1997). In the 1996 paper, the authors report that spatial patterns result from the non-linear coupling of the orthogonal components (x and y) of the 2-D trajectories produced by the index finger. The modulation of the relative phase, the relative amplitude, and the frequency ratio between the components engender the ensemble of the observed behaviors. Moreover, the model explains the passage from one pattern to a more stable pattern when a critical constraint such as movement speed is increased. As handwriting is a trajectory formation process, it should conform to this model, at least qualitatively, that is, it should show similar basic, global dynamic properties.

Assuming that handwriting dynamics is that of the non-linear coupling between two theoretical oscillatory components, a sound experimental strategy is in the first place to identify attractors in graphic patterns. [1] Handwriting behavior should exhibit only a limited number of preferred relative phase and/or amplitude ratios, the

[1] It is important to make it clear that, in this work, the two coupled oscillators are an abstract representation (see Section 4) as opposed to a more applied model à *la* Hollerbach, where the oscillators stand for actual, if simplified, movements of the hand joints. A mapping of theoretical oscillators onto biomechanical components is an expected development of this study.

parameters deemed to determine the ensemble of all possible trajectories. In order to reveal such underlying coordination dynamics, a paradigm established by Yamanishi, Kawato, and Suzuki (1980), developed by Tuller and Kelso (1985), and coined *scan* by Zanone and Kelso (1992) was adopted. By probing the entire space of relative phase (or amplitude), which gives rise to all possible coordination patterns, attractors are detected by comparing the produced to the required performance and by analyzing its variability. The rationale is straightforward. If a required relative phase corresponds to a preferred coordination pattern (viz. an attractor of the spontaneous coordination dynamics), then performance is accurate and stable. If the task does not coincide with a stable state, then performance is biased in the direction of the closest attractor and is more variable. Thus, attractors are easily identified in the plot of the constant error (CE), as a function of the required relative phase, say, from 0° to 180°: There is a characteristic negative slope at relative phases attracting nearby values. Note that the standard deviation (SD) at these values should be minimal as well. In order to explore graphic-like skills, the scanning method in our experiment consisted of performing various basic handwritten trajectories (lines and ellipses) corresponding to variations of either the relative phase or the relative amplitude between the two orthogonal oscillatory components. Under the hypothesis that several stable states define the underlying coordination dynamics (a regime called multistability), it follows that these shapes corresponding to preferred coordination patterns should be performed with higher accuracy and lower variability. The goal of the following experiment is thus to determine which patterns are the most stable and accurate.

2. Method

2.1. Participants

Six unpaid right-handed participants (five male, one female), aged 22 and 23, volunteered for the experiment.

2.2. Task

Participants were required to reproduce various ellipsoid shapes, displayed on a digitizing tablet, using an attached stylus. Participants were seated in an adjustable chair with both arms resting on the table where the tablet was inserted. Two sets of thirteen shapes, generated according to Eq. (1), were presented, ranging from a 2 cm long line to a circle 2 cm in diameter, going through several ellipses of varying eccentricities. The two sets differed relative to their basic orientation, diagonal or upright, corresponding to modulations of relative phase (RP) and relative amplitude (RA), respectively. For any given trial, the 13 shapes of one of the two sets appeared successively on the screen of the digitizing tablet. The task was to trace in superposition of the shape appearing on the screen of the digitizing tablet. Each shape stayed on the screen for seven seconds before changing into the following shape. The instructions were to be as accurate as possible at a constant, spontaneous speed and to always maintain contact between the stylus and the writing surface throughout a trial.

Fig. 1 presents the manipulation of the relative phase between the two orthogonal oscillators. It varied from 0° to 180° by 15° steps, producing a sequence of ellipsoids ranging from a right-slanted (0°) to a left-slanted straight (180°) segment, for a constant amplitude ratio of 1 ($A_x = A_y$). The task in question was called a *scan of relative phase* (RP).

Fig. 2 presents the manipulation of the relative amplitude between the two orthogonal oscillators. Thirteen relative amplitudes were obtained by diminishing the amplitude of one oscillator (from $A_x = 6$ to $A_x = 0$) in six equal steps, while maintaining the amplitude of the other oscillator at its maximal value ($A_y = 6$) and the relative phase constant at 90°. The same procedure was carried out after exchanging the oscillators. This produced a sequence of ellipsoids ranging from a vertical segment to a horizontal segment. This task was called a *scan of relative amplitude* (RA). Note that, in order to have a better understanding of the results, the relative amplitude variable was treated in terms of relative phase after an appropriate rotation of the coordinate axes (for more details, see Section 2.5 and Appendix A). The ensuing correspondence between relative amplitude and relative phase for each shape is given in Fig. 2.

For both scans, two additional variables were manipulated: (a) the direction of rotation of the motion used to trace the ellipsoids, clockwise (CW) and counterclockwise (CCW); and (b) the direction of progression of the shape parameters within a trial: from a right-slant (0°) to a left-slant (180°), or vice versa, for the RP scan, and from vertical (RA = 6:0) to horizontal (RA = 0:6), or vice versa, for the RA scan. Within a trial, the direction of rotation as well as the direction of progression was constant. Direction of rotation for any given trial was indicated to the participant before the trial.

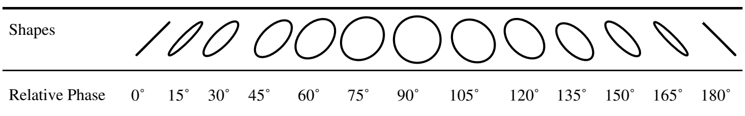

Fig. 1. Scan of relative phase (RP). Mapping of the diagonal shapes with their expression in relative phases. The diameter is 2 cm wide.

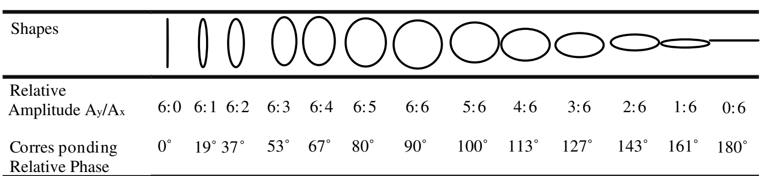

Fig. 2. Scan of relative amplitude (RA). Mapping of the upright shapes with their expression in amplitude ratios and in relative phases. Maximal amplitude, noted 6, corresponds to a length of 2 cm.

2.3. Procedure

The experiment involved, for each participant, two sessions carried out on two consecutive days. Each session lasted about 90 min. and was devoted to either the RP or the RA scan, the order of which was counterbalanced among participants. For each scan, a total of six repetitions was performed for each experimental condition, with a random assignment of each trial within a session. At the beginning of the first session, participants were familiarized with the apparatus and the task. They were asked to draw the circular shape belonging to both sets (RP = 90° and RA = 6:6) for 30 s. The position of the wrist on the tablet was recorded, so that it could be set similarly in every subsequent trial.

2.4. Apparatus

The 13 shapes were displayed consecutively for 7 s each at the center of a 21×15 cm backlit digitizing tablet, which was inserted in a table of adjustable height facing the participants. As soon as the stylus touched the tablet, the x and y coordinates (accuracy = 0.5 mm ± 0.02) of the performed trajectories were digitized at 100 Hz, fed back on-line for display on top of the current model shape on the tablet, and stored for later processing.

2.5. Data processing

Relative phase between the oscillatory components was calculated in degrees as a point-estimate (see Zanone & Kelso, 1992, for details) twice per cycle, at the maximal and minimal excursions of both time series, and then averaged over the entire trial.

Relative amplitude involved more processing. In order to compare the results of both scans and obtain a coherent measure of performance in terms of over- and underestimation, we rotated by 45° the original orthogonal x–y referential (see Appendix A for details of the procedure). Thus, it was possible to express all the RA requirements as various RP ranging from 0° to 180° (see Fig. 2 for explicit values). It is important to note that, as rotation of the coordinate axes maintains the topological and metric properties of the trajectories, the results, in particular the stability regime of the underlying dynamics, are comparable whether they express relative amplitude or relative phase.

For each requirement of each trial in the RP and RA scans, both expressed now in terms of relative phase (i.e., in degrees), we computed the mean Constant Error CE (the difference between the performed and required RP), the Absolute Error AE (its unsigned value), the associated Standard Deviation SD, and the average Frequency F (in Hz). If CE provides a good estimation of the location of attractive states in the underlying coordination dynamics, AE is a fair measure of accuracy, because it avoids the canceling out of the over- and underestimation in the vicinity of the attractors. SD provides an assessment of performance stability. Note that the error we are analyzing here is a temporal error (the difference between the phase relationship actually produced by the two oscillators and the required phase relationship) and not a measure of the spatial accuracy of the reproduced shapes. From now on, accuracy refers only to such temporal accuracy.

3. Results

A first analysis was performed in order to get a general appreciation of how good performance was in producing the various shapes or patterns for relative phase (RP) and relative amplitude (RA) scans. A 13(Pattern) × 2(Progression) × 2(Rotation) × 6(Trial) ANOVA with repeated measures on all factors was carried out on the mean AE and the SD of RP, and on the frequency *F*. Results showed that no main effect or interaction of Trial was detected for accuracy and variability measures (AE and SD), while there was a significant effect on the frequency *F* for the RA scan ($F(5,55) = 46.05$, $p < 0.0001$). This indicates that even when they realized the task more swiftly with practice, participants did not decrease accuracy, as they might have according to a classical speed-accuracy tradeoff, thereby complying with the requirement of reproducing the shapes with maximal precision.

From now on, the Trial factor will be removed from the following analyses of the RP and RA scans. We shall present results from 13(Pattern) × 2(Progression) × 2(Rotation) ANOVAs with repeated measures on all factors conducted on EA, SD, and *F*. A finer analysis of the significant effects will be provided by Newman–Keuls post-hoc tests with a threshold set at $p < 0.05$.

3.1. Scan of relative phase

3.1.1. Analysis of CE

In order to verify if subjects are producing the required phase relationship, or are under- or over-estimating it, we use the CE. Fig. 3A presents the results of the RP scans, averaged across participants and trials. The upper curve displays the mean

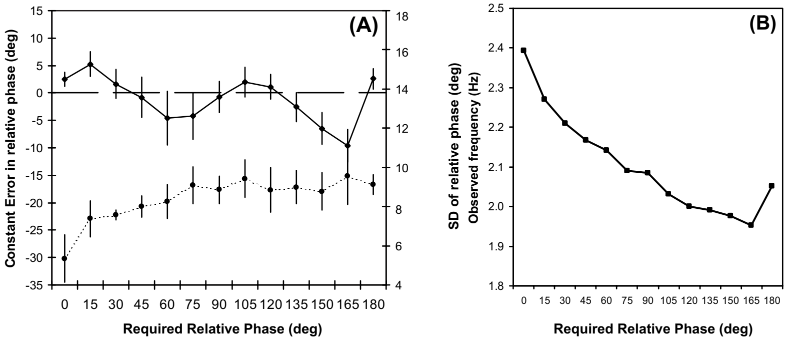

Fig. 3. Scan of RP. Panel A: CE (solid line) and SD (dotted line) of produced relative phase, as a function of the required relative phase. A negative error means that the performed phase relation was underestimated, and conversely. Vertical bars correspond to inter-subject SD. Panel B: Mean cycling frequency for each required relative phase.

CE of RP (left ordinate), as a function of the required RP (viz. the required shape; cf. Fig. 1). Note that for the sake of clarity data have been pooled over progression (0°–180° and vice versa).

Regarding mean CE of RP (top curve), there was a best match between the performed and required RP for 0°, 30°/45°, 90°/120°, and 180°. Moreover, two negative slopes are noticeable about 30°/45° and 120°: performed patterns in the vicinity of these values were systematically over- or underestimated, suggesting attraction. For example, when a 15° pattern was required, participants tended to produce a phase relationship close to 30°, while when a 60° pattern was required, the performed phase relationship was underestimated in the direction of the same 30°. A similar phenomenon occurred at about 120° of RP.

Fig. 4, which presents for each participant the mean CE as a function of the required RP, indicates that features suggesting attraction toward 0°, 30°/45°, 120°, and 180° are manifest for five out of six participants. Thus, negative slopes about 30°/45° and about 105°/130° are present for all participants, with a variation of ±15°, except for S4, where no negative slope shows up for 30°/45°, but rather toward 90°, if in a weaker fashion.

Let us examine closely the results as a function of the progression, from 0° (right-slanted shapes) to 180° (left-slanted shapes), or vice versa. Fig. 5 displays the mean CE of RP (solid curves, left ordinate) and the associated SD (dotted curves, right ordinate), as a function of the required RP, plotted separately for the two directions of progression. For mean CE, when the scan initiated with 0° (left curves), a first negative slope extended from 0° to 60°, crossing the abscissa at about 15°/30°, and a

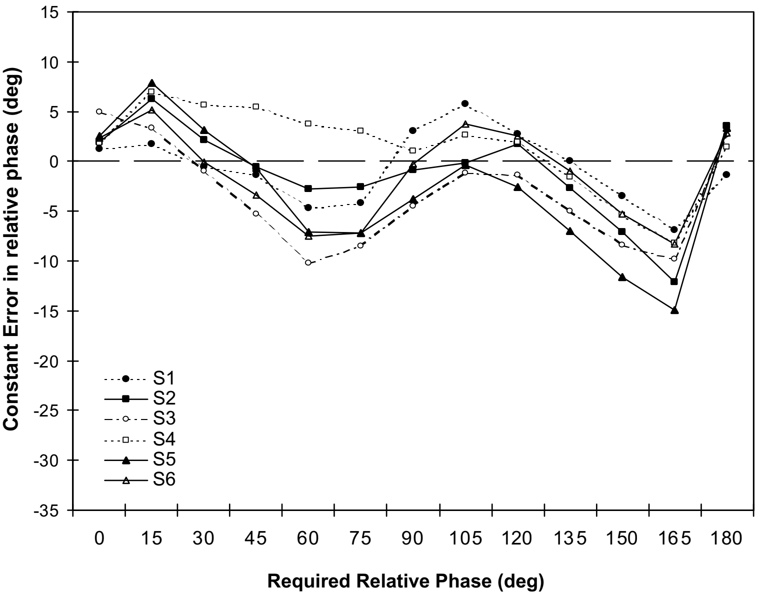

Fig. 4. Scan of RP. Mean CE of produced relative phase, as a function of the required relative phase for each participant.

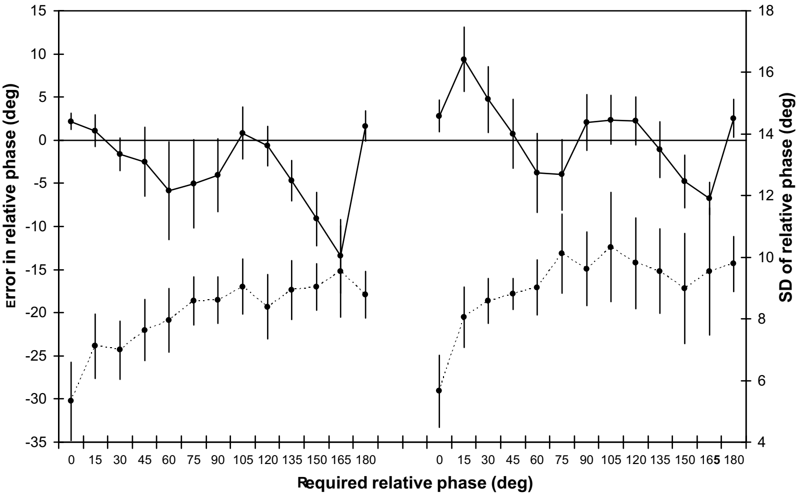

Fig. 5. Scan of RP. Left-hand curves: CE (solid line) and SD (dotted line) as a function of required relative phase for the 0°–180° progression. Right-hand curves: CE (solid line) and SD (dotted line) as a function of required relative phase for the 180° to 0° progression. Vertical bars correspond to inter-subject SD.

second zero-crossing occurred between 105° and 120°. When the scan started with 180° (right curves), zero-crossings were located about 120°/135° and 45°. Thus, the transitions from one shape to another did not occur at the same values depending on which shape was to be executed initially. In terms of variability, the 0° pattern was performed most stably irrespective of the direction of progression.

3.1.2. Analysis of AE

In order to substantiate these findings, a 13(Pattern) × 2(Progression) × 2(Rotation) ANOVA with repeated measures on all factors was performed on AE and SD of RP, and frequency F. Regarding AE, the analysis revealed main effects of Pattern ($F(12,420) = 37.03$, $p < 0.001$) and Progression ($F(1,35) = 4.98$, $p < 0.05$), and a Pattern × Progression interaction ($F(12,420) = 28.35$, $p < 0.0001$). Post-hoc analyses indicated that, overall, the 0° and 180° patterns exhibited a smaller AE than the others, and that AE was significantly reduced at about 30°/45° and 105°–135°. Still, for the progression from 0° to 180°, the 0°, 15°, 105°, and 180° patterns showed lowest AE, whereas for the 180°-to-0° progression, the 0°, 45°, 105° to 135°, and 180° patterns, the performed phase relationship was the closest to the required phase relationship. Results for AE corroborated those for CE, suggesting that 0° and 180° were produced most precisely, while there was a fairly good performance at about 30°/45° and 105°/135°. These tendencies interacted with the Rotation effect, since the ANOVA detected a main Rotation effect ($F(1,35) = 6.06$, $p < 0.02$), a Pattern × Rotation interaction ($F(12,420) = 6.73$, $p < 0.0001$), and a Rotation × Progression × Pattern interaction ($F(12,420) = 3.52$, $p < 0.0001$). This three-way interaction indicated that four of the 13 patterns exhibited a smaller AE for the CCW than for the CW rotation.

3.1.3. Analysis of SD

Regarding SD of RP, the bottom curve in Fig. 3A indicates that the variability of 0° RP was lower than all others. Note that variability was also generally lower for all the ellipses oriented toward the right (between 15° and 60°) than for those oriented toward the left (105°–180°). Among the latter, however, recall that shapes corresponding to 120° and 180° of RP were performed with higher (temporal) accuracy.

The ANOVA revealed a main Pattern and a main Progression effect ($F(12,420) = 41.60$, $p < 0.0001$ and $F(1,35) = 14.77$, $p < 0.001$, respectively) and a Pattern × Progression interaction ($F(12,420) = 3.79$, $p < 0.001$). The 0° pattern was the least variable, irrespective of the rotation and progression. Moreover, performance was less variable in the 0° to 180° progression than the other way round, except for the 100°, 113°, 127°, 161°, and 180° patterns, which were more variable when the scan started at 0°. Post-hoc analyses indicated that the 0° pattern was more stable than any other pattern, while the 15°, 30°, and 45° patterns were significantly less variable than all the patterns between 75° and 180°. Thus this analysis established a hierarchy among the patterns in terms of stability: 0° is more stable than 15°/45°, more stable than 75°/180°. In other words, the 0° pattern was produced most stably, and the right-slanted shapes were performed with more stability than the left-slanted shapes. These data, however, showed a significant Rotation effect and a Rotation × Pattern interaction ($F(1,35) = 6.52$, $p < 0.001$ and $F(12,420) = 2.55$, $p < 0.01$, respectively). The two-way interaction manifested that all trajectories were performed with a smaller variability in the CW than in the CCW rotation, except for the 165° and 180° patterns, as shown by post-hoc analyses.

3.1.4. Analysis of F

Regarding frequency F displayed in Fig. 3 panel B, the ANOVA detected a main Pattern and a main Rotation effect ($F(12,420) = 50.78$, $p < 0.0001$ and $F(1,35) = 15.72$, $p < 0.0001$, respectively) and a Pattern × Progression interaction ($F(12,40) = 4.52$, $p < 0.0001$). Post-hoc analyses showed that F decreased from 0° to 165° (from 2.4 to 2.0Hz) and that all patterns were performed at higher speed in the 0°-to-180° progression, except for 165°. Interestingly, the most accurate and least variable patterns, namely, 0°, 30°, and 45°, were also performed the most rapidly. Finally, the ANOVA also detected a Rotation × Pattern and a Rotation × Progression × Pattern ($F(12,420) = 9.25$, $p < 0.0001$ and $F(12,420) = 4.18$, $p < 0.0001$, respectively). This reflects the fact that the 15° and 30° patterns were performed at a significantly higher frequency than the other patterns in the CCW than the CW rotation in the 0°–180°.

3.2. Scan of relative amplitude

Analyses similar to those of the relative phase presented above were carried out for the relative amplitude: scrutinizing CE of RA should reveal the presence of attractors, while analyzing AE, SD and F should provide additional information.

3.2.1. Analysis of CE

Fig. 6A presents the results of the relative amplitude scan averaged across participants and trials. From now on, RA will be presented in terms of RP because all relative amplitudes have been converted to relative phases (for example 6:0 and 1:6 ratios became 0° and 161°; see Fig. 2 for the correspondences). Thus, the upper curve plots the CE of RP as a function of the required RP. CE exhibited two negative slopes between 37° and 53° and between 127° and 143°. Additionally, CE was low at 0° and 180°, as well as at about 80°.

Fig. 7 displays the same results on an individual basis. All participants showed a dynamical landscape comparable to the average landscape shown in Fig. 6, except for S1 and S6 who did not exhibit signs of attraction to 37°.

Let us now consider the effect of the direction of progression in the relative amplitude scan, from a vertical line (RA = 6:0, corresponding to 0° of RP) to horizontal (RA = 0:6, corresponding to 180°) shapes, and vice versa. Fig. 8 presents the mean CE (top solid curves) and the associated SD (bottom dotted curves) for each RP requirement as a function of progression. For a progression from vertical to horizontal shapes (viz. 0° to 180°), there were two negative slopes: The first extended from 0° to 53° and crossed the abscissa between 19° and 37°, while the second extended from 100° to 161° and crossed about 127°. In contrast, for the reverse progression, the evolution of CE was similar to that shown in Fig. 6, with two negative slopes that crossed the abscissa near 53° and near 143°.

In a fashion similar to that of the relative phase scan, a 13(Pattern) × 2(Progression) × 2(Rotation) ANOVA with repeated measures on all factors was performed on AE, SD of RP, and frequency F for the relative amplitude scans.

3.2.2. Analysis of AE

Regarding AE of RP, a general index of performance (temporal) accuracy, the analysis revealed a main effect of Pattern ($F(12, 420) = 14.22$, $p < 0.001$) and a

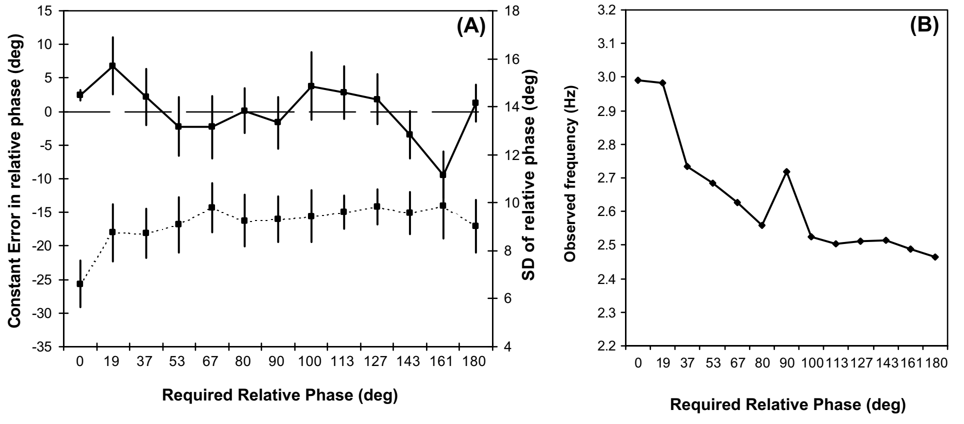

Fig. 6. Scan of RA. Panel A: CE (solid line) and SD (dotted line) of produced relative phase as a function of the required relative phase. Vertical bars correspond to inter-subject SD. Panel B: Mean cycling frequency for each required relative phase.

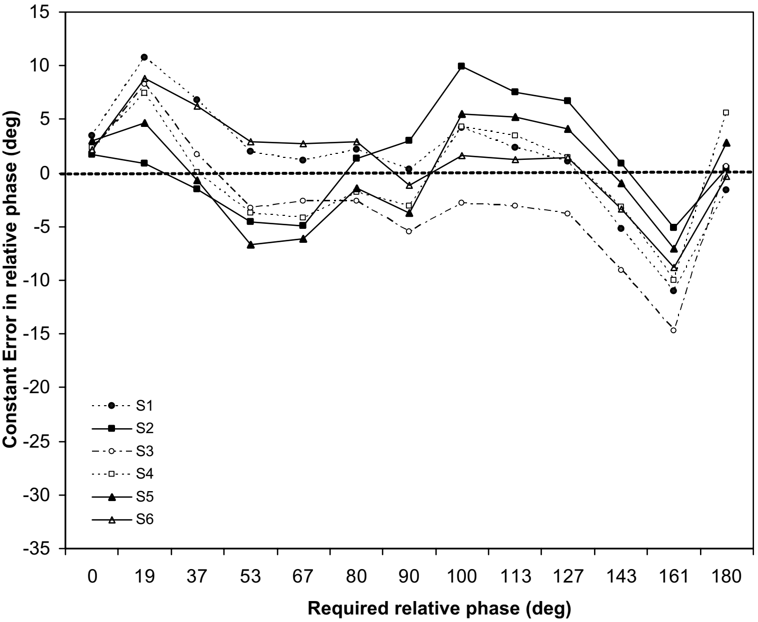

Fig. 7. Scan of RA. CE of relative phase, as a function of the required relative phase for each participant.

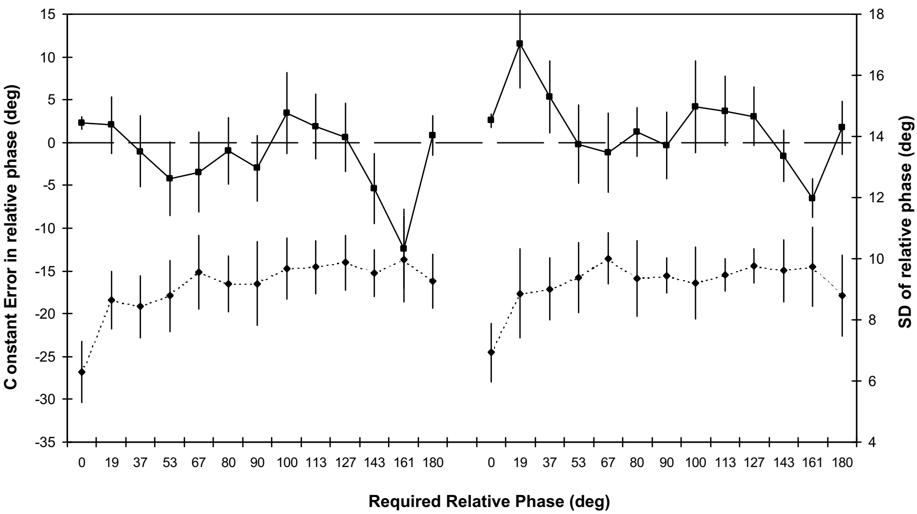

Fig. 8. Scan of RA. Left-hand curves: CE (solid line) and SD (dotted line) as a function of required relative phase for the 0° to 180° progression. Right-hand curves: CE (solid line) and SD (dotted line) as a function of required relative phase for the 180° to 0° progression. Vertical bars correspond to inter-subject SD.

Pattern × Progression interaction ($F(12, 420) = 17.81$, $p < 0.0001$). The interaction captured the fact that in the 0° to 180° progression (vertical to horizontal), the 0°, 19°, 37°, 80°, 127°, and 180° patterns were most accurate, whereas in the reverse progression (180° to 0°, viz. horizontal to vertical), the 53° pattern replaced the 19° and 37° patterns. Note that for both progressions, the 0° and 180° were clearly performed precisely. Results for AE suggest then four preferred patterns in terms of precision. Post-hoc analyses confirmed that the 0°, 80° and 180° patterns were produced with the lowest AE. Additionally, 0° was significantly more accurate than 180°. Within the remaining patterns, AE was markedly lower about 37° and 143°. Irrespective of the progression, the analyses corroborated the findings pertaining to CE: Attractive patterns (see Figs. 6 and 7) exhibited also the largest accuracy.

3.2.3. Analysis of SD

Fig. 6A (bottom curve) shows that SD was minimal at 0° and tended to decrease toward 180°. Taken together with the CE results (see above description), these findings suggest attractive states at about 0°, 37°, 130°, and 180°.

The ANOVA detected a main Pattern effect ($F(12, 420) = 28.74$, $p < 0.0001$) and a Pattern × Progression interaction ($F(12, 420) = 2.45$, $p < 0.001$). Whereas the 0° pattern was least variable for both progression, patterns between 0° and 90° (oriented vertically) were less variable than the others in the vertical-to-horizontal progression (0° to 180°), but more variable in the horizontal-to-vertical progression. Post-hoc contrasts indicated (a) that 0° was significantly more stable than all the others; (b) that the 19°–37° patterns were less variable than those between 67° and 161°; and (c) a significant decrease in variability between 163° and 180°. To sum up, the 0° pattern was the most stable pattern, while the others could be ranked SD-wise following 0° < 19° −37° < 180° < other patterns.

3.2.4. Analysis of F

Finally, as can be seen in Fig. 6B, the results concerning frequency F were in line with those of the relative phase scan. The ANOVA revealed mean Pattern and Progression effects ($F(12, 420) = 70.39$, $p < 0.001$ and $F(12, 420) = 36.92$, $p < 0.001$, respectively) and a Pattern × Progression interaction ($F(12, 420) = 12.96$, $p < 0.001$). Mean frequency decreased significantly from 0° ($F = 2.99$ Hz) to 180° ($F = 2.46$ Hz) with a marked intermediate increase about 90°, progression from vertical to horizontal (0° to 180°) was performed at higher speed than in the reverse order, and the decrease in frequency from 0° to 180° was significantly steeper when the initial pattern was 180°.

4. Discussion

This study aimed to detect and localize preferred coordination patterns, or attractors, of the dynamics underlying graphic skills, through a procedure that required performing several ellipsoid shapes corresponding to varying relative phases or relative amplitudes between two orthogonal oscillators. We shall first discuss the results relative to the orientation of the ellipses and then interpret them in terms of underlying spontaneous coordination dynamics.

In each scan (i.e., relative phase and relative amplitude), four patterns stood out in terms of accuracy. The first two were the extreme 0° and 180° RP patterns, corresponding to orthogonal lines of diagonal or upright orientations. The last two were relative phases at about 45° and 120° (i.e., 30°–45° and 105°–120° for the RP scan, 53° and 127° for the RA scan) and defined ellipses of intermediate eccentricities, close to those reported in the recent study by Dounskaia et al. (2000). Their work indicated that performing a perfectly circular trajectory was quite difficult and that increasing speed made the RP decrease, so that ellipses were performed instead of circles. Our results provide additional information. First, such a bias to intermediate eccentricities is not only induced by increasing speed as shown by Dounskaia et al. (2000), but also exists at spontaneous speed: It manifests preferred, spontaneous coordination tendencies, or attractors of underlying coordination dynamics. Second, for both scans and both progressions, the spontaneous dynamics is very comparable, with preferred, attractive ellipses corresponding to practically identical RP values. That similarity of the attractor layout across scans (RP versus RA) points to the abstract nature of the dynamics, as they remain basically invariant through the 45° rotation, a rotation that must entail a notable change in the actual implementation of the end-effector components. Such independence of coordination dynamics from the effectors, a property already reported in Kelso and Zanone (2002), may be at the origin of the well-known motor equivalence characterizing handwriting (Merton, 1972).

Such preferred coordination patterns also stuck out in terms of speed of execution. Our results on the relative phase scan indicated that the most stable patterns (0°, 30° and 45°) were performed at the highest frequency. Conversely, in spite of the explicit requirement to maintain frequency constant, participants spontaneously diminished frequency for left-slanted shapes, which were also less stable. Similarly, for the relative amplitude scan, the vertically oriented 0° and 19° to 37° patterns were performed faster and with less variability than their horizontally oriented counterparts.

To sum up the results, not only were preferred coordination patterns performed stably, but also with higher temporal accuracy and speed. These findings suggest that handwriting movements are not homogeneously difficult: A pattern is intrinsically difficult as a function of its stability. [2] Preferred, stable patterns are the easiest, so they can be performed with temporal accuracy and rapidity. Less stable patterns are more difficult so that, when frequency is to be maintained constant in accordance with the task requirements, like in the present situation, they are executed less precisely. Finally, if the patterns are quite unstable, hence very difficult, not only temporal accuracy is poor, but also speed is reduced. Consequently, a uniform increase of either the speed (e.g. to take notes) or the accuracy (e.g., to write an

[2] Such a link between difficulty and stability was also shown in studies by Temprado and colleagues on attention (see Monno, Temprado, Zanone, & Laurent, 2002, for a review): The less stable patterns are more difficult to execute and require more attention to be sustained than the more stable patterns.

important official letter) constraint should lead to differential effects on the handwriting performance. Assuming that handwriting is formed as a succession of patterns such as the ones tested here, increasing uniformly a constraint, the writing speed for example, should lead to a loss of (temporal) accuracy, the magnitude of which being different for each pattern. One should thus be able to predict that some letters, corresponding to combinations of less stable patterns, will be more variable than other letters that correspond to combinations of more stable patterns. How this difference in temporal stability translates into actual shapes of the letter remains to be studied. Furthermore, additional legibility constraints will most probably impose that certain individual features of the letters vary systematically less than others.

Our findings are also consistent with reported effects of handwriting orientations (Meulenbroek & Thomassen, 1991). Graphic shapes are more precise and stable when they are tilted to the right (viz 0°) as opposed to the left (180°), as well as when they are oriented vertically as opposed to horizontally. Similar results were described in a less systematic study by van Sommers (1984) and were attributed to biomechanical properties of the joints involved in handwriting.

The main goal of the present study was to draw the blueprints for a dynamical account of graphic skills, in particular handwriting. Several findings are quite supportive of such a view. On the one hand, we reported ample evidence of a dynamic interplay between existing patterns and the required task based on stability properties, in particular, attraction. Not only were less stable patterns performed with lesser precision and speed, but they were also biased toward the closest preferred, more stable pattern, as shown by a systematic constant error (i.e., over- and underestimation for lower and higher values, respectively). Attraction of nearby trajectories to stable states is a hallmark of dissipative dynamical systems, a general class of systems to which all biological, if not all natural systems do belong (e.g., Kelso, 1995). Moreover, in both relative phase and amplitude scans, the direction in which the progression through the task requirements was carried out led to different values of relative phase at which the transitions between stable patterns occurred: They always occurred later in one direction than in the converse (cf. Figs. 5 and 7). In dynamical system theory, such a resistance to change is called hysteresis, a clear sign of non-linearity. On the other hand, in both relative phase and amplitude scans, we failed to show attractive properties for the most stable 0° and 180° patterns through a negative slope of constant error. On the contrary, once the task required a pattern, say 15°, other than the preferred one currently performed, say 0°, the system "escaped" the latter to adopt the "next available" stable state, say 45° (cf. Fig. 1 or Fig. 6). Such a behavior due to the stepping in the task requirement has also been documented and discussed for bimanual coordination (Zanone & Kelso, 1992). Conceptually, a study by Tuller, Case, Ding, and Kelso (1994) on categorization in speech perception showed a similar early change from one stable state to the other, coined "enhanced contrast". Theoretically, such an inclination to "anticipate" the passage to another stable state with the slightest modification in the parameter value is an "anti-hysteresis", another sign of non-linearity of the underlying dynamics. Thus, the spontaneous changes in behavior induced by modifying the graphic shapes to reproduce exhibit all the types of transitions expected in non-linear dynamical systems. These findings expand the idea of two combined orthogonal oscillators (Hollerbach, 1981; Singer & Tishby, 1994) in showing that those are actually coupled in a *non-linear* fashion, which has important behavioral and theoretical consequences.

In spite of the large number of possible temporal relationships between the two oscillators involved in handwriting, only a few are spontaneously adopted. Strikingly, the dynamics underlying handwriting conforms to that found in trajectory formation (Buchanan et al., 1996), where the execution of 2-D spatial trajectories corresponding to 8, 0, and 1 shapes and the transitions among those proved to be governed by the dynamics of non-linear coupled oscillators. Thus, graphic skills, and particularly handwriting, alike all periodic interlimb coordination, basically rely on the dynamics of non-linearly coupled (non-linear) oscillators (Haken et al., 1985).

Accordingly, the most stable, hence precise and swift patterns turned out to be the in-phase (0° of relative phase between the oscillators) and the anti-phase (180°) synchronization between the oscillators, although results were more mitigated concerning the latter. Moreover, both relative phase and amplitude scans indicated that besides these "classical" expected spontaneous patterns, handwriting resorts to two other modes of stable coordination at about 50° and 120°, corresponding to ellipses of intermediate eccentricities. The emergence of multistable (i.e., quadristable) dynamics suggests that handwriting involves a more sophisticated coordination than, say, bimanual coordination *á la* Kelso (1984). One should keep in mind, for example, that the necessity to produce a sizeable number of clearly differentiated signs (for example, letters) imposes specific constraints on the end-effector, in particular the production of curved trajectories corresponding to relative phases other than the 0° and 180° stable modes. In line with a self-organizational view, coordination dynamics adapted to handwriting are brought about by the interplay of task and intrinsic constraints. Studies on learning (Zanone & Kelso, 1992) demonstrated that novel, previously unstable coordination patterns may be stabilized permanently with practice, thereby being incorporated into the initial spontaneous coordination dynamics. Given its rich dynamics, it comes then as no surprise that handwriting is probably the skill that is practiced by humans for the longest time.

The aim of our work is to reframe the handwriting behavior in light of non-linear dynamics concepts. As a first step, we have shown that we can describe our experimental data in terms of two non-linearly coupled oscillators. Undoubtedly, we now need to bridge the gap between a conceptualization of the temporal coordination of two theoretical oscillators and actual handwriting. As a first approximation, one can conceive of the theoretical oscillators as being two abstract components that capture at a higher level the entire neurobiomechanical degrees of freedom involved in the task, a notion not foreign to the classic synergies *á la* Bernstein (1967). The mapping of the coordination of these higher level oscillators onto the lower level end-effector oscillators is most probably neither a simple, nor a trivial, one-to-one relationship. For example, consider the biomechanical properties of the writing hand: As opposed to the theoretical oscillators we have proposed in the present experiment, the actual fingers and hand oscillators are asymmetric, that is, their respective dynamics (e.g., eigenfrequencies) are not identical. Further, in the present study, the orthogonal oscillating components are positioned arbitrarily in the x–y referential of the writing surface (here, the tablet). Previous studies, looking at the actual shapes produced, showed that the biomechanical axes are not orthogonal (e.g., Dooijes, 1983). However, one should keep in mind that planar transformation aiming to align the theoretical and actual oscillators preserves topological properties. Therefore, the same multistable coordination dynamics would be found, albeit with slight scalar modifications (i.e., changes in the very values at which stable states are identified).

The first evidence for a dynamic view of handwriting reported here yields a new perspective on motor behavior in graphic skills. In the literature, two conceptualizations prevail. On the one hand, a "bottom–up" approach provides a good account of the kinematic and dynamic properties of the trajectories (Edelman & Flash, 1987; Viviani & Cenzato, 1985; Viviani & Terzuolo, 1982, 1983; Wada, Koike, Vatikiotis-Bateson, & Kawato, 1995). On the other hand, a "top–down" approach describes the cognitive processes involved in handwriting and dissociates behavior in functional units (Van Galen, 1991). None of those, however, says anything about two basic features of handwriting: (a) the systematic deformations of the trace due to increasing constraints, such as speed or stress, and (b) the coarticulation of two successive strokes or units in time and space, in which any single graphic element is pro- and retroactively altered with respect to the following and preceding unit. Any comprehensive theory of handwriting must spell out the rules that govern both phenomena.

A key to answer the above issues is the fact that when a non-specific constraint, such as frequency, is increased, thereby destabilizing the system, spontaneous patterns disappear in the inverse order of their respective stability, for the more stable the patterns are, the more they withstand perturbations impinging on the system. Empirical evidence for such a self-organizational phenomenon has been reported in bimanual coordination, where the in-phase pattern substitutes the anti-phase pattern (Kelso, 1984), or in trajectory formation (Buchanan et al., 1996), where the 8, 0, and 1 patterns are successively adopted with increasing movement speed. Our basic tenet regarding handwriting is that the building blocks, the functional units, are trajectories defined in terms of stable phase relationships between the oscillators, corresponding to (segments of) ellipsoids of a given eccentricity. A necessary step is to determine more precisely the stability of such spontaneous coordination patterns, thereby determining how fast and easy the passage from one trajectory to another is: The more stable the pattern is, the shorter the switching time (Scholz & Kelso, 1990) and the easier the transition. It should then be possible to predict how handwriting deteriorates with increasing constraints: Progressively, only the most stable patterns are producible, leading to a simplification of the script characterized by a smaller set of performed trajectories and swifter transitions.

To sum up, both performance and degradation in handwriting find a unifying concept with the notion of stability, a hallmark in dynamical systems theories. Moreover, the conflict raised by Irigoin (1990) between simplification and differentiation may well pertain to the availability of a limited set of stable graphic shapes. A subtle interplay must be achieved between the tendency to perform the most stable patterns in order to simplify handwriting movements and the necessity to use less stable patterns in order to increase the total number of available shapes, and thus legibility. Therefore, the theoretical and methodological tools of a dynamical systems approach may reveal common principles of formation of basic graphic shapes, their co-articulation, as well as their learning and degradation.

Acknowledgments

We are indebted to Philippe Truillet from the University of Paul Sabatier in Toulouse for his invaluable help in this study. IS is supported by a research award from the French Ministry of Education. This paper was written in part while SA and PGZ were on leave at the Center for Complex Systems and Brain Sciences, Florida Atlantic University, USA.

Appendix A

Assuming that periodic drawing is produced by approximately sinusoidal orthogonal oscillations, each component can be described by the following equation:

$$\begin{aligned} x(t) &= A_x \cos(\omega_x(t-t_0)+\phi_x) \\ y(t) &= A_y \cos(\omega_y(t)+\phi_y) \end{aligned} \tag{A.1}$$

where A_x and A_y are horizontal and vertical amplitude, ω_x and ω_y are frequency and ϕ_x and ϕ_y are the phase of each oscillator. We note $A = A_x/A_y$, the ratio between each amplitude component. First, to have a similar measure for the both tasks, we made a rotation of axes references for the relative amplitude scan. The new coordinates x' and y' in the rotated reference system were calculated for each trial and each subject with the formulas:

$$\begin{aligned} x'(t) &= x\cos\theta + y\sin\theta \\ y'(t) &= -x\sin\theta + y\cos\theta \end{aligned} \tag{A.2}$$

where $\theta = 45°$.

Secondly, the correspondence between each required amplitude relative in the initial coordinate system xy (required AR) and required relative phase pattern in this oblique system $x'y'$ were computed. New parameters of the shapes were calculated in the oblique coordinate system with the formulas:

$$\begin{aligned} x'(t) &= \frac{\sqrt{1+A^2}}{\sqrt{2}}\cos(\omega t+\varphi 1) \\ y'(t) &= \frac{\sqrt{1+A^2}}{\sqrt{2}}\cos(\omega t+\varphi 2) \end{aligned} \tag{A.3}$$

where $Ax = \frac{\sqrt{1+A^2}}{\sqrt{2}} = Ay$, A correspond to the required AR pattern in the xy coordinate system, and $\varphi 1$ and $\varphi 2$ are the phase of x' and y' respectively. Then, the relative phase can be calculated for each required shape as:

$$\varphi = ar\cos\frac{1-A^2}{1+A^2} \quad \text{or} \quad \varphi = ar\sin\frac{2A}{1+A^2} \tag{A.4}$$

References

Bernstein, N. (1967). *The coordination and regulation of movements*. Oxford: Pergamon Press.

Buchanan, J. J., Kelso, J. A. S., & Guzman, G. C. de (1997). The self-organization of trajectory formation: I. Experimental evidence. *Biological Cybernetics, 76*, 257–273.

Buchanan, J. J., Kelso, J. A. S., & Fuchs, A. (1996). Coordination dynamics of trajectory formation. *Biological Cybernetics, 74*, 41–54.

Dooijes, E. H. (1983). Analysis of handwriting movements. *Acta Psychologica, 54*, 99–114.

Dounskaia, N., van Gemmert, A. W. A., & Stelmach, G. E. (2000). Interjoint coordination during handwriting-like movements. *Experimental Brain Research, 135*, 127–140.

Edelman, S., & Flash, T. (1987). A model of handwriting. *Biological Cybernetics, 57*, 25–36.

Guzman, G. C. de, Kelso, J. A. S., & Buchanan, J. J. (1997). Self-organization of trajectory formation: II. Theoretical model. *Biological Cybernetics, 76*, 275–284.

Haken, H., Kelso, J. A. S., & Bunz, H. (1985). A theoretical model of phase transitions in human hand movements. *Biological Cybernetics, 51*, 347–356.

Hollerbach, J. M. (1981). An oscillation theory of handwriting. *Biological Cybernetics, 39*, 139–156.

Hulstijn, W., & van Galen, G. P. (1983). Programming in handwriting: Reaction time and movement time as a function of sequence length. *Acta Psychologica, 54*, 23–49.

Irigoin, J. (1990). L'alphabet grec et son geste des origines au IXième siècle après J.-C. In C. Sirat, J. Irigoin, & E. Poulle (Eds.), *L'écriture: le cerveau, l'oeil et la main*. *Bibliologia*, 10, 299–305.

Kelso, J. A. S. (1984). Phase transitions and critical behavior in human bimanual coordination. *American Journal of Physiology, Regulatory, Integrative and Comparative Physiology, 15*, R1000–R1004.

Kelso, J. A. S. (1995). *Dynamic patterns: The self-organization of brain and behavior*. Cambridge, MA: MIT Press.

Kelso, J. A. S., & Jeka, J. J. (1992). Symmetry dynamics of human multilimb coordination. *Journal of Experimental Psychology: Human Perception and Performance, 18*, 645–688.

Kelso, J. A. S., & Schöner, G. (1987). Toward a physical (synergetic) theory of biological coordination. *Springer Proceedings in Physics, 19*, 224–237.

Kelso, J. A. S., & Zanone, P. G. (2002). The coordination dynamics of learning and transfer across effector systems. *Journal of Experimental Psychology: Human Perception and Performance, 28*, 776–797.

Merton, P. (1972). How we control the contraction of our muscles. *Scientific American, 226*, 30–37.

Meulenbroek, R. G. J., & Thomassen, A. J. W. M. (1991). Stroke-direction preferences in drawing and handwriting. *Human Movement Science, 10*, 247–270.

Monno, A., Temprado, J. J., Zanone, P. G., & Laurent, M. (2002). The interplay of attention and bimanual coordination dynamics. *Acta Psychologica, 110*, 187–211.

Schmidt, R. C., Carello, C., & Turvey, M. T. (1990). Phase transitions and critical fluctuations in the visual coordination of rhythmic movements between people. *Journal of Experimental Psychology: Human Perception and Performance, 16*, 227–247.

Scholz, J. P., & Kelso, J. A. S. (1990). Intentional switching between patterns of bimanual coordination is dependent on the intrinsic dynamics of the patterns. *Journal of Motor Behavior, 22*, 98–124.

Singer, Y., & Tishby, N. (1994). Dynamical encoding of cursive handwriting. *Biological Cybernetics, 71*, 227–237.

Teulings, H. L. (1996). Handwriting movement control. In S. W. Keele, & H. Heuer (Eds.), *Handbook of perception and action* (*Vol. 2*) (pp. 561–612). London: Academic Press.

Teulings, H. L., Thomassen, A. J. W. M., & Maarse, F. J. (1989). A description of handwriting in terms of main axes. In R. Plamondon, C. Y. Suen, & M. L. Simner (Eds.), *Computer recognition and human production of handwriting* (pp. 193–211). Singapore: World Scientific.

Teulings, H. L., Thomassen, A. J. W. M., & van Galen, G. P. (1986). Invariants in handwriting: The information contained in a motor program. In H. S. R. Kao, G. P. V. Galen, & R. Hoosain (Eds.), *Computer recognition and human production of handwriting* (pp. 305–316). Amsterdam: North-Holland.

Thomassen, A. J. W. M., & Meulenbroek, R. G. J. (1998). Low-frequency periodicity in the coordination of progressive handwriting. *Acta Psychologica, 100*, 133–144.

Tuller, B., & Kelso, J. A. S. (1985). Environmentally-specified patterns of movement coordination in normal and split-brain subjects. *Experimental Brain Research, 75*, 306–316.

Tuller, B., Case, P., Ding, M., & Kelso, J. A. S. (1994). The non-linear dynamics of speech categorization. *Journal of Experimental Psychology: Human Perception and Performance, 20*(1), 3–16.

Van der Plaats, R. E., & van Galen, G. P. (1990). Effects of spatial and motor demands in handwriting. *Journal of Motor Behavior, 22*(3), 361–385.

Van Galen, G. P. (1991). Handwriting: Issues for a psychomotor theory. *Human Movement Science, 10*, 165–191.

Van Sommers, P. (1984). *Drawing and cognition: Descriptive and experimental studies of graphic production processes*. New York: Cambridge University Press.

Viviani, P., & Cenzato, M. (1985). Segmentation and coupling in complex movements. *Journal of Experimental Psychology: Human Perception and Performance, 11*, 828–845.

Viviani, P., & Terzuolo, C. A. (1982). Trajectory determines movement dynamics. *Neurosciences, 7*, 431–437.

Viviani, P., & Terzuolo, C. A. (1983). The organization of movement in handwriting and typing. In: B. Butterworth (Ed.), *Language production* (*vol. II.*): *Development, writing and other language processes* (pp. 103–146). New York: Academic Press.

Wada, Y., Koike, Y., Vatikiotis-Bateson, E., & Kawato, M. (1995). A computational theory for movement pattern recognition based on optimal movement pattern generation. *Biological Cybernetics, 73*, 15–25.

Yamanishi, J., Kawato, M., & Suzuki, R. (1980). Two coupled oscillators as a model for the coordinated finger tapping by both hands. *Biological Cybernetics, 37*, 219–225.

Zanone, P. G., & Kelso, J. A. S. (1992). Evolution of behavioral attractors with learning: Non equilibrium phase transition. *Journal of Experimental Psychology: Human Perception and Performance, 18*, 403–421.